

## Modeling of Solar Radiation on Part Shaded Walls

Mohammad Nikian\*

Faculty of Mechanical Engineering, Takestan Branch, Islamic Azad University, Takestan, Iran

<b>Keywords:</b>	<b>Abstract</b>
Solar radiation, Cooling load calculation, Shaded wall.	The program described in this research produces average solar radiation on walls after shading effects have been considered by numerical analysis methods. After extensive testing, it is hoped that, by integrating a version of this program into existing systems, a more realistic solar heat gain may be obtained for the site. As a result of this, more economical systems can be installed that will operate at more efficient levels benefiting both the user (capital and running costs) and the supplier (more competitive quotes). The program developed appeared to successfully deal with the majority of shading cases that are liable to be met in load estimating. The program is quick and reasonably accurate (to within about 2% of hand calculation) with as few as 100 data points.

AZ	solar azimuth
AL	solar altitude
Dpx	x-co-ord of a point to be analyzed
Dpy	y-co-ord of a point to be analyzed
Dpz	z-co-ord of a point to be analyzed
SVX	solar vector x component
SVY	solar vector y component
SVZ	solar vector z component
SOX	shading wall origin point x
SOY	shading wall origin point y
SOZ	shading wall origin point z
WAHX	horizontal shading wall length (x direction)
WAHY	horizontal shading wall length (y direction)
WAV	vertical height of a shading wall
ITH	Total intensity on horizontal surfaces (w/sqm)
IDH	Direct intensity on horizontal surfaces (w/sqm)
Idh	Diffuse intensity on horizontal surfaces (w/sqm)
IDV	Direct intensity on vertical surfaces (w/sqm)
ITV	Total intensity on vertical surfaces (w/sqm)

### 1. Introduction

To calculate zone loads, all load components must be considered separately as internal or external loads. A typical set of loads might be: Solar gain, Glass transmission, Wall transmission, Roof

\* Corresponding Author :

E-mail, [m.nikiyan@tiau.ac.ir](mailto:m.nikiyan@tiau.ac.ir) – Tel, (+98) 2835270131 – Fax, (+98) 2835270165

Received: 12 February 2015 ; Accepted: 19 April 2015

transmission, Lighting, Other electrical, People and Cooling infiltration. Each of these may be further sub-divided into sensible and latent to determine the overall cooling or heating air supply required. The magnitude of the components varies and peak at different times [1]. For outside wall zones, the solar gain is often a very significant component (for instance up to 50% [2], depending on the position of the sun and the size of the glazing. The large importance of the solar gain component is rarely matched by the sophistication of the calculations used. Thus, zone peaks and subsequent sizing may be highly influenced by errors generated within calculations .

The estimation of heating and cooling loads on a building prior to the installation of an air conditioning system is both complex and liable to large errors. Most systems are now sized by commercial load estimating computer programs. Large errors can be sliced from solar gain estimations if shading is included. Load estimating programs for determining the size of air conditioning programs take into account many heat gain sources. One of the most significant external gains arises due to solar radiation on walls and windows. The calculation of solar radiation is often unrealistically high due to the lack of consideration given to shading on the external walls.

Most commercial load estimating programs were found to be without building shading equations. Two programs investigated, ([3] and [4]) were found to identify shading associated with external wall features. A third program was investigated in detail is an advanced load estimating program that was devised by the National Bureau of Standards at the Centre for Building Technology [5].

## **2. Objectives**

To investigate an effective method for the calculation of solar radiation on a building walls the position and orientation of other interfering walls and building. The output should be calculated with a view to using it as an input to additional program modules (for instance - to investigate dynamic heat transfer through the building surface).

The method should be fast, accurate and verifiable. It should also be easy to use but flexible. The project should relate to programs presently in use and how the new programs might integrate into existing methods.

### **2.1. Solar Position Definitions**

The sun position is given by two angles; the solar azimuth and the solar altitude. Their definitions are as follows:

**a: The solar altitude (al)**

The angle a direct ray from the sun makes with the horizontal at a particular place on the surface of the earth (Figure 1).

$$\sin(a_1) = \sin(D) \times \sin(LAT) + \cos(D) \times \cos(LAT) \times \cos(h) \quad (1)$$

**b: The solar azimuth (z)**

This is the angle the horizontal component of a direct ray from the sun makes with the true NorthSouth axis. It is expressed as an angular displacement through 360 degrees from true North (in the clockwise direction) (Figure 2).

To calculate both of the above angles, it is necessary to know the sun's position relative to the plane of rotation of the Earth (declination) and the position of the site on the surface of the Earth (latitude). Also the local time must be used for the calculation of the 'sun-time'. This eliminates need for the longitude of the site to be entered. The following definitions list this required information:

$$\tan(z) = \frac{\sin(h)}{\sin(LAT) \times \cos(h) - \cos(LAT) \times \tan(D)} \quad (2)$$

**c: Declination (d)**

This is the angular displacement of the sun from the plane of rotation of the Earth's equator. The value of the declination will vary throughout the year between  $-23.5^\circ$  and  $+23.5^\circ$  because the Earth is tilted at an angle of about  $23.5^\circ$  to the axis of the plane in which it orbits the sun. Figure 3 shows the relationship.

$$D = 23.47 \sin(360 \times (284 + N) / 365) \quad (3)$$

Where  $N$  is the day number (January 1st=1)

**d: Latitude (LAT)**

An angular displacement above or below the plane of the equator measured from the centre of the earth, gives the latitude of a site (shown in Figure 4).

**e: Sun time (T)**

This is the time in hours before or after noon.

**f: Hour angle (h)**

The angular displacement of the sun from afternoon:

$$h = (360/24) \times T \quad (4)$$

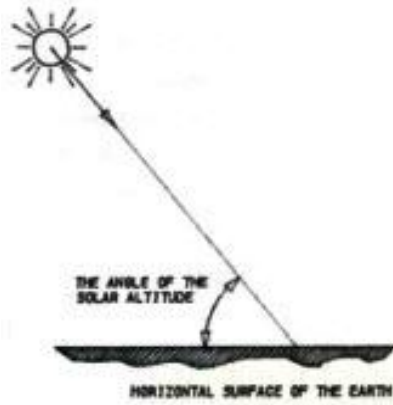


Figure 1. Soalr altitude

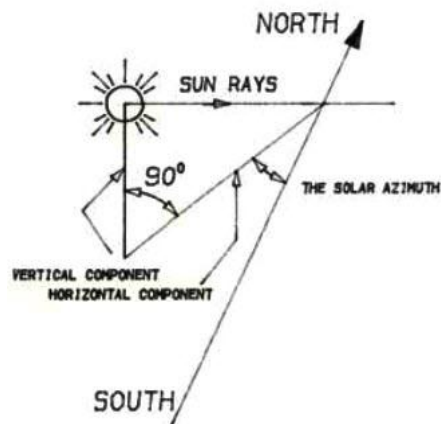


Figure 2. Solar Azimuth

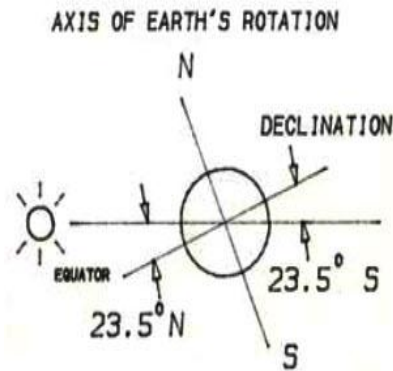


Figure 3. Declination

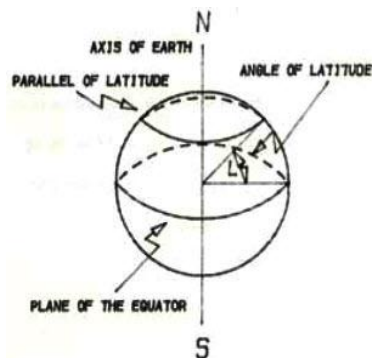


Figure 4. Latitude

### 3. Shading by Walls and Buildings

The main purpose of the program is to provide a reliable and fairly quick method of building shading analysis. The three dimensional geometry is fairly simple but difficult to generalize the formula into one simple case. To establish whether shading of a point (on the wall to be analyzed) occurs, the geometry of the site and surrounding buildings require to be known. The definition of the walls and buildings on the site are covered in detail in the program section. However, the walls may be assumed to be rectangular and perpendicular to the ground and to be completely opaque.

#### 3.1. Solar Radiation

##### 3.1.1. Direct Radiation

The intensity of direct radiation on a vertical surface is easily calculated if the beam radiation 'N' is known. For a wall-solar angle of 'WAZ' and a solar altitude 'al' the intensity of the direct vertical component is given by:

$$IDV = N \times \cos(\alpha_1) \times \cos(WAZ) \quad (5)$$

##### 3.1.2. Diffuse Radiation

Direct radiation entering the Earth's atmosphere is subject to scattering to create 'sky radiation' or 'diffuse radiation'.

The processes by which this occurs can be split into four categories:

- I. Radiant energy scattered by atmospheric molecules of ideal gas (eg. nitrogen, oxygen)
- II. Scattering due to presence of water vapor
- III. Selective absorption of water vapor
- IV. Scattering by dust particles

Sky radiation can't be assigned a specific direction (and hence no shadows are cast by it). The intensity of sky radiation is usually much less than that for direct radiation but cannot be ignored. The quantity of sky radiation varies with atmosphere's variation of gas composition, water vapor content, and dust content. It also varies as the solar altitude changes.

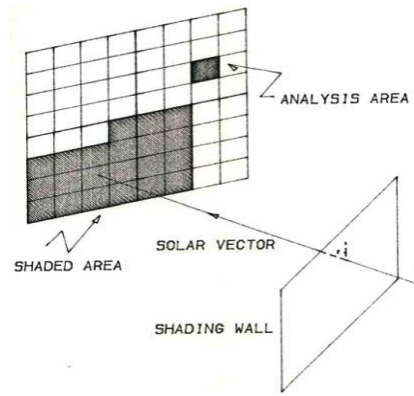


Figure 5. Solar vector and shading wall intersection

### 3.2. Derivation of the Shading Equations

The derivation of the following formulae is complicated somewhat by the problems of 1/0 errors (caused by  $\tan(A/B)$  when  $B=0$ ;  $1/\cos(A)$ ,  $1/\sin(A)$  when  $A=0$ ). The formulae have to be rearranged to cater for such eventualities and a test routine at the start of this section would then be required to ensure the correct equations are used. The diagram (Fig. 5) shows the typical situation and the definition of the terms is given below:

#### General Equation:

$$Dpx + k_1(SVX) = SOX + k_2(WAHX) \quad (6)$$

$$Dpy + k_1(SVY) = SOY + k_2(WAHY) \quad (7)$$

$$Dpz + k_1(SVZ) = SOZ + k_3(WAV) \quad (8)$$

Where  $k_1, k_2, k_3$  are three unknown constants to be found

#### Solution to General Equation

Rearranging general equations (6), (7) and (8):

$$k_2(SVX) = (SOX - Dpx) + k_1(WAHX) \quad (9)$$

$$k_2(SVY) = (SOY - Dpy) + k_1(WAHY) \quad (10)$$

$$k_2(SVZ) = (SOZ - Dpz) + k_3(WAV) \quad (11)$$

Multiply equation (9) by  $(SVY/SVX)$ :

$$k_2(SVY) = (SVY/SVX) \times (COXED) + k_1(SVY/SVX) \times (WAHX) \quad (12)$$

Subtracting by equation (10):

$$0 = (SVY/SVX) \times (COXED) - (SOY - Dpy) + k_1(((SVY/SVX) \times WAHX) - WAHY) \quad (13)$$

So,

$$k_1 = \frac{(((SOY - Dpy) - (SVY/SVX) \times (COXED)))}{(((SVY/SVX) \times WAHX) - WAHY)} \quad (14)$$

And from equation (10) we have:

$$k_2 = \frac{((SOY - Dpy) - k_1(WAHY))}{SVY} \quad (15)$$

Also using equation (11) gives:

$$k_3 = \frac{((SVZ/SVY)((SOY - Dpy) + k_1(WAHY))) - (SOY - Dpy)}{WAV} \quad (16)$$

Substituting for k<sub>1</sub>:

$$k_3 = \frac{WAHY \times ((SOY - Dpy) - (SVY/SVX) \times (COXED))}{WAV \times ((SVY/SVX) \times (WAHX - WAHY))} + \frac{((Dpz - SOZ) + (SVY/SVX)(SOY - Dpy))}{WAV} \quad (17)$$

### 3.3. Spatial Cases

This is a list of some possible problems to be considered when the general equation is to be solved. All those listed below indicate 1/0 errors may arise during calculation:

- 1 WAHX=0: wall is aligned N-S or S-N
- 2 WAHY=0: wall is aligned E-W or W-E
- 3 SVX=0: sun is due south (midday)
- 4 SVY=0: sun is due east or due west
- 5 SVZ=0,<0: sunrise/sunset and during night

### 3.3.1. Solution for Special Equations:

Special cases require modification to the general solution. These will be dealt with in order:

#### Case 1 (WAHX=0)

$$k_1 = \frac{[(SVY/SVX) \times (COXED) + (Dpy - SOY)]}{WAHY} \quad (18)$$

$$k_2 = \frac{(COXED)}{SVX} \quad (19)$$

Hence:

$$k_3 = \frac{[(SVZ/SVY)(COXED)] + (Dpz - SOZ)}{WAV} \quad (20)$$

#### Case 2 (WAHY=0)

As long as the wall vector WAHY is not zero, the expressions derived for the general equation can be used with SVY=0. This produces:

$$k_1 = \frac{[(SVY/SVX) \times (SOY - Dpy) + (Dpx - SOX)]}{WAH} \quad (21)$$

$$k_2 = \frac{(SOY - Dpy)}{SVY} \quad (22)$$

$$k_3 = \frac{[(SVZ/SVY)(SOY - Dpy)] + (Dpz - SOZ)}{WAV} \quad (23)$$

#### Case 3 (SVY=0)

As long as the wall vector WAHY is not zero, the expression derived for the general equation can be used with SVY=0. This produces:

$$k_1 = \frac{(SOY - Dpy)}{SVY} \quad (24)$$

$$k_2 = \frac{[(COXED) + (WAHX/WAHY)(Dpy - SOY)]}{SVX} \quad (25)$$

$$k_3 = \frac{(SVZ \times WAV)[(COXED) + (WAHX/WAHY)(Dpy - SOY)]}{SVX} - \frac{(SOZ - Dpz)}{WAV} \quad (26)$$



Case 4 (SVX=0)

Rearranging the general equations and assuming that SVY is not zero:

$$k_1 = \frac{(Dpx - SOX)}{WAHX} \quad (27)$$

$$k_2 = \frac{[(SOY - Dpy) + (WAHX/WAHY)(Dpx - SOX)]}{SVY} \quad (28)$$

$$k_3 = \frac{SVZ[(SOY - Dpy) + (WAHX/WAHY)(Dpx - SOX)]}{SVY \times WAV} - \frac{(SOZ - Dpz)}{WAV} \quad (29)$$

Case 5 (SVZ<0 or SVZ=0)

This can easily be detected before any calculation is carried out and the physical interpretation of this is that the sun has not risen. The solar data for this case is thus considered to be zero and no further shading calculation are therefore required.

**3.3.2. Calculation of SVX, SVY, SVZ:**

The calculation of these vectors may be easily achieved by the use of the altitude and azimuth angles as derived earlier:

$$\begin{aligned} SVX &= \cos(a_1) \sin(az) \\ SVY &= \cos(a_1) \cos(az) \\ SVZ &= \sin(a_1) \end{aligned} \quad (30)$$

**3.4. Solar Data Generation**

This method uses clear sky solar data generated by sinusoidal equations that have been fitted to experimental data. Assuming that the solar angles are already known:

*D* = Declination, *L* = Latitude, *A* = Solar altitude, *Z* = Solar azimuth, *S* = Face orientation of window analyzed; Then the sun normal intensity (*N*) w/sqm at each hour is:

$$\begin{aligned} N &= 1074.16 \times \sin(A) + 1980.060 \times \sin(3A) + 70.1766 \times \sin(5A) + 30.3902 \times \sin(7A) + \\ &13.3842 \times \sin(9A) + 5.59234 \times \sin(11A) + 2.93048 \times \sin(13A) + 0.606472 \times \sin(15A) \end{aligned} \quad (31)$$

It is then necessary to correct the intensities by applying an altitude correction factor (*K<sub>a</sub>*) for sites with an elevation 300m or greater above sea level:

$$K_a = 1.02 + 0.00002 \times \text{elevation} + 0.00005 \times \text{elevation} \times (1/\sin(A)) \quad (32)$$

$$Idh = 121.649 \times \sin(A) + 14.7575 \times \sin(3A) + 7.72576 \times \sin(5A) + 3.47353 \times \sin(7A) + 2.22222 \times \sin(9A) + 0.52539 \times \sin(11A) + 0.52164 \times \sin(13A) + 0.1311 \times \sin(15A) \quad (33)$$

For clear sky conditions:

$I=1$  Overall radiation factor,  $K_c=0.95$  Direct radiation factor,  $K_r=0.2$  Ground reflection factor,  $c=0$  Cloudiness,  $fc_1=1$  Cloudiness factor

$$ITH = Ka \times I \times (K_c \times fc_1 \times IDH \times Idh) \quad (34)$$

$$IDV = N \times \cos(A) \times \cos(Z - S) \quad (35)$$

$$ITV = K_a \times I \times (K_c \times fc_1 \times IDV + 0.5 \times Idh + 0.5 \times K_r \times ITH) \quad (36)$$

Window is specified on, the window will receive only diffuse radiation and hence:

$$IDV = 0, IDH = 0$$

#### 4. Numerical Analysis

For a general case solution to all the possible geometrical problems posed by shading, analytical techniques would be cumbersome and very complex, and not necessarily quicker or more accurate. Analytical techniques could be used to find the shading boundaries of the problem; numerical analysis is especially useful for calculating the area that is shaded.

Numerical techniques require the splitting up of an area to be analyzed into smaller areas. Each area is assigned a central point where the equations for that area are evaluated. It is assumed that the conditions at this point are then valid for the rest of the area. Thus the problem is broken down into discrete point analysis rather than the calculation of a continuum. This is more straightforward, and is relatively easy to convert into a computing sequence.

In previous part, , a set of equations were derived to establish whether shading of a particular point (Dpx, Dpy, Dpz) occurs due to another arbitrarily positioned wall at a certain time. It can be seen that it is fairly easy to incorporate the testing of a series of data points into the analysis of a single larger area. Thus the style of the program emerges: The testing of these shading equations on sets of data point co-ordinates throughout the building will establish the fractions of the walls that are shaded.

### 5. Results and Discussions

This section gives an example of which aims to demonstrate the advantages of the program. This is shown as plan view of the site, a listing of the results and graphs of the solar radiation in watts per square meter on the windows analyzed. The building have been analyzed for two time intervals, January 800hrs - 1600hrs and June 800hrs - 1600hrs. This helps show the annual variations of sun position and the consequent variations in the shading patterns. The significance of the shading is very evident.

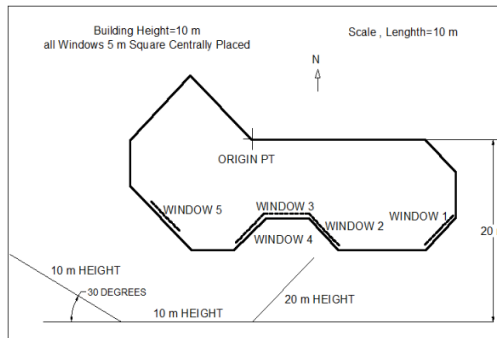


Figure 6. Plan view of example

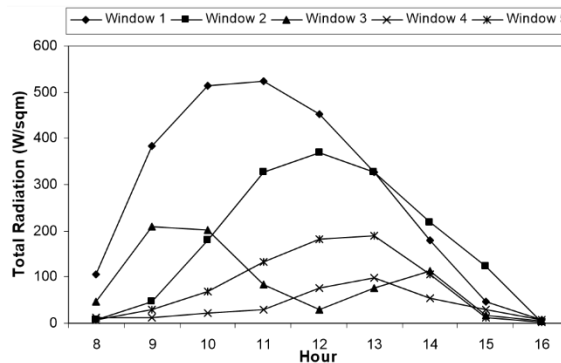
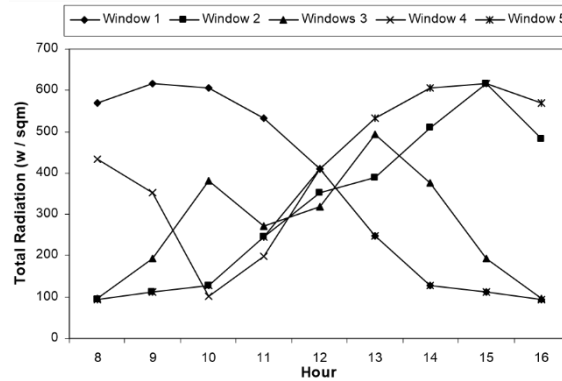


Figure 7. Total Radiation for 5 Windows in Example Building Time Interval of January 800hrs – 1600 hrs

Table 1. Calculation Results for Window 1 in Example Building in January

Shading Radiation Data for Window 1 (100 Data Points)						
Month	Time (hr)	ITV (w/sqm)	IDV (w/sqm)	%Direct Rad.	Total (w/sqm)	Total Watts
Jan	8:00	105.71	103.41	100	105.71	2643.1
Jan	9:00	384.52	373.09	100	384.52	9614.2
Jan	10:00	515.07	493.91	100	515.07	12878.4
Jan	11:00	524.28	493.67	100	524.28	13108.8
Jan	12:00	451.73	413.97	100	451.73	11294.8
Jan	13:00	327.00	286.01	100	327.00	8176.0
Jan	14:00	178.76	139.89	100	178.76	4469.5
Jan	15:00	45.94	16.70	100	45.94	1148.8
Jan	16:00	7.47	0.00	0	7.47	186.8



**Figure 8.** Total Radiation for 5 Windows in Example Building  
Time Interval of June 800hrs – 1600 hrs

**Table 2.** Calculation Results for Window 1 in Example Building in June

Shading Radiation Data for Window 1 (100 Data Points)

Month	Time(hrs)	ITV (w/sqm)	IDV (w/sqm)	%Direct Rad.	Total (w/sqm)	Total Watts
Jun	8:00	569.17	499.74	100	569.17	14231.0
Jun	9:00	617.58	531.06	100	617.58	15441.5
Jun	10:00	604.68	503.12	100	604.68	15119.0
Jun	11:00	532.54	417.95	100	532.54	13315.4
Jun	12:00	408.94	284.46	100	408.94	10224.8
Jun	13:00	246.88	117.25	100	246.88	6172.7
Jun	14:00	126.72	0.00	0	126.72	3168.4
Jun	15:00	113.07	0.00	0	113.07	2827.2
Jun	16:00	94.41	0.00	0	94.41	2360.7

## 6. Conclusions

The overall objective of this research was to stress how important the effects of shading are on the incident solar radiation on the building (and consequently the cooling load on the air conditioning equipment).

The program described in this research produces average solar radiation on walls after shading effects have been considered by numerical analysis methods. After extensive testing, it is hoped that, by integrating a version of this program into existing systems, a more realistic solar heat gain may be obtained for the site. As a result of this, more economical systems can be installed that will operate at more efficient levels benefiting both the user (capital and running costs) and the supplier (more competitive quotes). The program developed appeared to successfully deal with the majority of shading cases that are liable to be met in load estimating. The program is quick and reasonably accurate (to within about 2% of hand calculations) with as few as 100 data points. The program apparently indicates the shading equations are correct together with the methodology behind their use.

## References

- [1] ASHRAE, ANSI/ASHRAE Standard 189.1: Standard for the Design of High-Performance Green Buildings Except Low-Rise Residential Buildings, ASHRAE, Atlanta, GA, (2011).
- [2] Calculation done using Carrier E20-II HAP (version 3.4), August, (2003).
- [3] CIBSE Heat Gains (version 2.13) program.
- [4] Carrier HAP (version 4.3) program.
- [5] Kasuda, T NBSLD: The computer program for heating and cooling loads, (1976).